Subsystems and Independence in Relativistic Microscopic Physics *

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Abstract

The analyzability of the universe into subsystems requires a concept of the "independence" of the subsystems, of which the relativistic quantum world supports many distinct notions which either coincide or are trivial in the classical setting. The multitude of such notions and the complex relations between them will only be adumbrated here. The emphasis of the discussion is placed upon the warrant for and the consequences of a particular notion of subsystem independence, which, it is proposed, should be viewed as primary and, it is argued, provides a reasonable framework within which to sensibly speak of relativistic quantum subsystems.

1 Introduction

Without the possibility of analyzing the universe into subsystems, it is hardly conceivable how the sciences could be carried out. Common experience certainly supports this possibility; however, common experience is neither quantum nor relativistic, so it is far from obvious whether one can sensibly speak of microscopic subsystems, despite the fact that much of science is carried out as if one could do so. Indeed, what appears to be straightforward in a classical, nonrelativistic setting turns out to be highly nontrivial and, to some degree, impossible in a relativistic quantum setting. However, it is not our purpose here to rehearse the well known controversies concerning the various kinds of nonlocality and interdependence of subsystems manifest in relativistic quantum theory (cf. [22, 23, 57, 65] for recent

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discussions and reviews). Instead, our intent is to indicate how, in spite of these, one can still speak meaningfully of subsystems in relativistic quantum theory. Although no new theorems will be proven in this paper, we shall draw together results scattered through many highly technical papers to make a coherent case for this claim. The technicalities will be minimized as much as possible, however.

What is a subsystem of a relativistic quantum system? We shall not answer this question here. Indeed, as explained in Section 7, even after the analysis carried out below, there are other subtle matters to deal with before such a definition can be attempted. But whatever a subsystem is, it is not merely a spatially distinguished portion of the full system. To be conceptually most useful, a subsystem should be an identifiable component of the system which can subsist independently of the other subsystems comprising the system, *e.g.* it can be suitably screened off from the other subsystems and studied experimentally without their influence. The analyzability of the universe into subsystems therefore requires a concept of the "independence" of the subsystems, of which the relativistic quantum world supports many distinct notions which coincide or are trivial in the classical setting. The complex relation between these notions will only be adumbrated here; the emphasis will be placed upon the warrant for and the consequences of a particular notion of subsystem independence, which, it is proposed, should be viewed as primary and which, it is argued, provides a reasonable framework within which to sensibly speak of relativistic quantum subsystems.

In order to formulate in a mathematically rigorous manner the notion of independent subsystems and to understand its consequences, it is necessary to choose a mathematical framework which is sufficiently general to subsume large classes of relativistic quantum models, is powerful enough to facilitate the proof of nontrivial assertions of physical interest, and yet is conceptually simple enough to have a direct, if idealized, interpretation in terms of operationally meaningful physical quantities. Such a framework is provided by algebraic quantum field theory (AQFT) and algebraic quantum statistical mechanics [1, 3, 4, 36], also called collectively local quantum physics, which is based on operator algebra theory, itself initially developed by J. von Neumann for the express purpose of providing quantum theory with a rigorous and flexible foundation [55, 56]. This framework is briefly described in the next section.

In Section 3 we discuss three of the many notions of independence which have been examined in the literature, indicating briefly their operational meaning and their logical interrelations. But what we regard as the operationally primary notion of independence — the split property — is initially discussed in Section 4. This property is strictly stronger than all those treated in Section 3. After the somewhat abstract discussion in Section 4, we present in Section 5 a number of equivalent characterizations of the split property which all have *operational* meaning. Further physically significant consequences of the split property are reviewed in Section 6 to buttress our contention that the split property should be viewed as the primary independence notion. Various aspects of the warrant for the split property are considered in Sections 4—6. Finally, in Section 7 we draw our conclusions and

indicate why the analysis of the notion of independent subsystems in relativistic quantum theory is far from complete.

2 Mathematical Framework

The operationally fundamental objects in a laboratory are the preparation apparata — devices which prepare in a repeatable manner the individual quantum systems which are to be examined — and the measuring apparata — devices which are applied to the prepared systems and which measure the "value" of some observable property of the system. The physical notion of a "state" can be viewed as a certain equivalence class of such preparation devices, and the physical notion of an "observable" (or "effect") can be viewed as a certain equivalence class of such measuring (or registration) devices [1,50]. In principle, therefore, these objects are operationally determined, albeit quite abstract.

It should be emphasized that these apparata can be effectively extremely small, as exemplified by atomic traps or atomic probes. Indeed, one probes the inner structure of protons and neutrons by collisions with other suitable elementary particles. It requires no undue stretch of the imagination to consider such collisions as being part of the chain of either the preparation or the measuring apparata. Admittedly, such apparata are theory–dependent, but both the design and the interpretation of *all* experiments are strongly theory–dependent. We therefore see no immediate obstacle to admitting the existence of apparata of submicroscopic extent. And, although it is possible in principle to derive quantum theory without any reference to microsystems by using only the description of macrosystems in terms of suitable state spaces [51], we shall also posit the existence of microsystems and accept that quantum theory describes (ensembles of) such systems.

In algebraic quantum theory, such observables are represented by self–adjoint elements of certain algebras of operators, either W^*- or C^*-algebras.[1] In this paper we shall restrict our attention primarily to concretely represented W^*-algebras, which are commonly called von Neumann algebras in honor of the person who initiated their study [56]. The reader unfamiliar with these notions may simply think of algebras $\mathcal{M}$ of bounded operators on some (separable) Hilbert space $\mathcal{H}$ (or see [41,42,67–69] for a thorough background). We shall denote by $\mathcal{B}(\mathcal{H})$ the algebra of all bounded operators on $\mathcal{H}$. Physical states are represented by mathematical *states* ϕ, *i.e.* linear, (norm) continuous maps $\phi : \mathcal{M} \to \mathbb{C}$ from the algebra of observables to the field of complex numbers which take the value 1 on the identity map I on $\mathcal{H}$ and are positive in the sense that $\phi(A^*A) \geq 0$ for all $A \in \mathcal{M}$. An important subclass of states consists of *normal states*; these are states such that $\phi(A) = \mathrm{Tr}(\rho A)$, $A \in \mathcal{M}$, for some *density matrix* ρ acting on $\mathcal{H}$, *i.e.* a bounded

[1] For some purposes a more general formulation of observable as a certain kind of positive operator valued measure is useful [24,40,50,57]. However, this more general class of observables is subsumed into the present setting if the measure takes its values in an algebra of the sort discussed here.

operator on $\mathcal{H}$ satisfying the conditions $0 \leq \rho = \rho^*$ and $\mathrm{Tr}(\rho) = 1$. A special case of such normal states is constituted by the *vector states*: if $\Phi \in \mathcal{H}$ is a unit vector and $P_\Phi \in \mathcal{B}(\mathcal{H})$ is the orthogonal projection onto the one dimensional subspace of $\mathcal{H}$ spanned by Φ, the corresponding vector state is given by

$$\phi(A) = \langle \Phi, A\Phi \rangle = \mathrm{Tr}(P_\Phi A)\,,\ A \in \mathcal{M}\,.$$

Generally speaking, theoretical physicists tacitly restrict their attention to normal states; for the purposes of this paper, it will suffice to do so as well.

In local quantum physics, one takes account of the localization of the observables from the very outset. Indeed, in the context of relativistic quantum field theory, since any measurement is carried out in a finite spatial region and with a finite elapse of time, for every observable A there must exist bounded (open, nonempty) spacetime regions $\mathcal{O}$ in which A can be localized.[2] We denote by $\mathcal{A}(\mathcal{O})$ the algebra generated by all observables localized in $\mathcal{O}$. Clearly, it follows that if $\mathcal{O}_1 \subset \mathcal{O}_2$, then $\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)$. This yields a net $\mathcal{O} \mapsto \mathcal{A}(\mathcal{O})$ of observable algebras associated with the experiment(s) in question. In turn, this net determines the smallest algebra $\mathcal{A}$ containing all $\mathcal{A}(\mathcal{O})$. The preparation procedures in the experiment(s) then determine states ϕ on $\mathcal{A}$, the global observable algebra.

The standard picture of a Hilbert space of state vectors familiar from von Neumann's formulation of nonrelativistic quantum mechanics is then recovered as follows. A state ϕ on $\mathcal{A}$ uniquely determines (up to unitary equivalence [41, 67]) the GNS representation $(\mathcal{H}_\phi, \pi_\phi, \Phi)$ of $\mathcal{A}$ with $\pi_\phi : \mathcal{A} \to \mathcal{B}(\mathcal{H}_\phi)$ a representation of $\mathcal{A}$ as a concrete algebra of bounded operators acting on a Hilbert space $\mathcal{H}_\phi$, $\Phi \in \mathcal{H}_\phi$ a cyclic vector for $\pi_\phi(\mathcal{A})$,[3] and, for all $A \in \mathcal{A}$,

$$\phi(A) = \langle \Phi, \pi_\phi(A)\Phi \rangle\,.$$

The conceptual and mathematical advantages which accrue to the use of this more general notion of state are manifold, but they will not play a role in this paper.

In the setting of relativistic quantum physics, it is argued that spacelike separated events must be, in some sense, independent. Indeed, if two events are spacelike separated, there exists an inertial reference frame in which they are *simultaneous*. Since events which happen simultaneously cannot reasonably influence each other, each must be independent of the other (though they may have a common causal antecedent). One way this independence is expressed in AQFT is through the property of *locality* (sometimes referred to as microcausality or Einstein causality), namely if $\mathcal{O}_1$ and $\mathcal{O}_2$ are spacelike separated regions, then $\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)' = \{B \in \mathcal{B}(\mathcal{H}) \mid AB - BA = 0\,,\ \text{for all} \quad A \in \mathcal{A}(\mathcal{O}_2)\}$. However,

[2] It is clear from operational considerations that one could not expect to determine a minimal localization region for a given observable experimentally. In [45] the possibility of determining such a minimal localization region in the idealized context of AQFT is discussed at length. In any case, from the remarks made above it is clear that such localization regions can be very small indeed. And in the idealized setting of quantum field theory, there are nontrivial algebras of observables associated with every nonempty region $\mathcal{O}$ — see *e.g.* [31].

[3] *i.e.* the set of vectors $\pi_\phi(A)\Phi$, $A \in \mathcal{A}$, is dense in $\mathcal{H}_\phi$

as shall be explained below, in most concrete models much stronger forms of independence are satisfied by observable algebras associated with spacelike separated regions.

It should be remarked that the word "locality" is used in at least two distinct ways in quantum theory. In nonrelativistic quantum theory, the nonlocality spoken of when referring to Bell's inequalities or entangled states in quantum information theory is a property of certain *states* on the observable algebras.[4] In relativistic quantum field theory (and quantum statistical mechanics), locality is a property of the *observable algebras*. These two properties are perfectly compatible with each other, as is evidenced by the fact that, quite generically, there exist pairs of local algebras in relativistic quantum field models for which Bell's inequalities are *maximally violated* in *all* normal states [63], so that all such states are maximally entangled (and therefore maximally "nonlocal") across such pairs, even though the algebras themselves satisfy locality.

3 Some Formulations of Subsystem Independence

There are various technical conditions used in algebraic quantum theory to formulate the notion of the independence of subsystems. This is only to be expected, since there are clearly different quantitative and qualitative aspects of such independence. The study of these formulations and their logical relations is therefore of some conceptual interest. In the context of classical mechanics or classical field theory, these notions are either trivial or mutually equivalent. However, in the quantum setting they are distinct and nontrivial. It is not our purpose here to review the multitude of notions which have been under discussion in the literature nor to explain their logical interrelationships (but see [32, 38, 58, 64] for such reviews). Instead we shall restrict our attention in this paper to just four of these. Two are commonly employed in the theoretical physics literature, though only in the context of a special case.

Surely the most familiar formulation of the independence of two subsystems with observable algebras $\mathcal{A}$ and $\mathcal{B}$, respectively, is that their observables be mutually commensurable (or "jointly measurable", "mutually coexistent", *etc.*):

Commensurability of Observables: $[A, B] = AB - BA = 0$, for all $A \in \mathcal{A}$, $B \in \mathcal{B}$, *i.e.* $\mathcal{A} \subset \mathcal{B}'$.

As this formulation is so familiar, we shall not elaborate upon its operational significance, though such a discussion can be found in [21, 50, 64]. We remark that in the setting alluded to above, where observables are modelled by positive operator valued measures, commutative observables are jointly measurable (or mutually coexistent) but the converse is false [21, 46, 47] for certain pairs or finite families of

[4] In the voluminous literature on Bell's inequality there are yet other uses of the word "local" whose relations seem to be uncharted.

observables. Nonetheless, when concerned with independence of *subsystems*, one is clearly interested in the joint measurability of *all* pairs of observables of the subsystems, and this *is* equivalent to $\mathcal{A} \subset \mathcal{B}'$ (see *e.g.* [54, 64]).

In [35] R. Haag and D. Kastler introduced a notion of subsystem independence they named statistical independence. This is the property that each of the two subsystems can be prepared in any state, independently of the preparation of the other system. In [64] this property was formalized for different classes of observable algebras (and the distinctions proved to be apposite). Since we are restricting ourselves to von Neumann algebras in this paper and are not trying to treat independence notions exhaustively, we shall only mention the property termed W^*-independence in [64].

Statistical Independence (W^*-Independence): Let $\mathcal{A}$ and $\mathcal{B}$ be von Neumann algebras acting on the Hilbert space $\mathcal{H}$. The pair $(\mathcal{A}, \mathcal{B})$ is statistically independent if for any normal state ϕ_1 on $\mathcal{A}$ and any normal state ϕ_2 on $\mathcal{B}$, there exists a normal state ϕ on $\mathcal{B}(\mathcal{H})$ such that $\phi(A) = \phi_1(A)$ and $\phi(B) = \phi_2(B)$ for all $A \in \mathcal{A}$, $B \in \mathcal{B}$.

One sees that if $\mathcal{A}$ and $\mathcal{B}$ represent the observable algebras associated with two subsystems, the statistical independence of the pair $(\mathcal{A}, \mathcal{B})$ can be loosely interpreted as follows: any two partial states on the two subsystems can be realized by a suitable preparation of the full system; no choice of a state prepared on one subsystem can prevent the other subsystem from being in any prescribed state.

It turns out that commensurability of observables and statistical independence are logically independent notions. Indeed, there exist von Neumann algebras which do not mutually commute and yet are statistically independent; and there exist mutually commuting von Neumann algebras which are not statistically independent — cf. [64] for a discussion, examples and further references.

The third notion of subsystem independence we shall consider here is of more recent origin [61] and is closely related to the fourth, and primary, notion to be discussed in the next section. A few preparations shall prove to be necessary.

We recall that a linear map $T\colon \mathcal{A} \to \mathcal{B}$ can be extended to a linear map $T_n\colon M_n(\mathcal{A}) \to M_n(\mathcal{B})$ (here $M_n(\mathcal{A})$ is the set of n by n matrices with elements from the algebra $\mathcal{A}$) by

$$T_n \begin{pmatrix} A_{11} & \dots & A_{1n} \\ . & . & . \\ A_{n1} & \dots & A_{nn} \end{pmatrix} = \begin{pmatrix} T(A_{11}) & \dots & T(A_{1n}) \\ . & . & . \\ T(A_{n1}) & \dots & T(A_{nn}) \end{pmatrix} .$$

T is said to be *completely positive* if T_n is positive for every $n \in \mathbb{N}$, *i.e.* T_n maps positive operators to positive operators. A completely positive map $T : \mathcal{A} \to \mathcal{A}$ satisfying $T(I) \leq I$ is called an *operation* [24, 44]. An operation T such that $T(I) = I$ is said to be *nonselective*. An operation T on a von Neumann algebra $\mathcal{A}$ is called *normal* if it is σ–weakly continuous (the natural topology associated with von Neumann algebras). A positive linear map $T : \mathcal{A} \to \mathcal{B}$ is *faithful* if $T(A) > 0$ whenever $\mathcal{A} \ni A > 0$.

Operations are mathematical representations of physical operations, *i.e.* physical processes which take place as a result of physical interactions with the quantum system. (For a more detailed discussion of operations see [44].) A state on $\mathcal{A}$ is a completely positive unit preserving map from $\mathcal{A}$ to $\mathbb{C}$. So, if ϕ is a state on $\mathcal{A}$, then

$$\mathcal{A} \ni A \mapsto T(A) = \phi(A)I \in \mathcal{A} \tag{3.1}$$

is a nonselective operation in the sense of the above definition, which is canonically associated with the state and which may be interpreted as the preparation of the system into the state ϕ. In fact, for any state ω and $A \in \mathcal{A}$ one has $\omega(T(A)) = \omega(\phi(A)I) = \phi(A)$. Further examples of operations are provided by measurements. In particular, if one measures a quantum system with observable algebra $\mathcal{B}(\mathcal{H})$ for the value of a (possibly unbounded) observable Q with purely discrete spectrum $\{\lambda_i\}$ and corresponding spectral projections P_i, then according to the "projection postulate" this measurement can be represented by the operation T defined as

$$\mathcal{B}(\mathcal{H}) \ni X \mapsto T(X) = \sum_i P_i X P_i \in \mathcal{B}(\mathcal{H}) \, . \tag{3.2}$$

T is a normal nonselective operation on $\mathcal{B}(\mathcal{H})$. In fact, K. Kraus proved that any normal operation T on $\mathcal{B}(\mathcal{H})$ must have the form

$$T(X) = \sum_i W_i^* X W_i \qquad\qquad \sum_i W_i^* W_i \leq I \, ,$$

where $W_i \in \mathcal{B}(\mathcal{H})$ [44]. It should also be mentioned that in quantum information theory, unit preserving completely positive linear maps $T : \mathcal{A} \to \mathcal{B}$ are called *channels* and are of central interest [43].

In the light of these considerations, the following generalization of statistical independence is natural. Once again, we restrict our attention to just one of the notions introduced in [61], in this case to what was termed there operational W^*-independence in the product sense. Note that it is not assumed that the algebras mutually commute.

Operational Independence (Operational W^*-Independence in the Product Sense): A pair $(\mathcal{A}, \mathcal{B})$ of von Neumann algebras is *operationally independent in* $\mathcal{A} \vee \mathcal{B}$, the smallest von Neumann algebra containing both $\mathcal{A}$ and $\mathcal{B}$, if any two (faithful) normal nonselective operations on $\mathcal{A}$ and $\mathcal{B}$, respectively, have a joint extension to a (faithful) normal nonselective operation on $\mathcal{A} \vee \mathcal{B}$ which factors across the pair; *i.e.* if for any two (faithful) normal completely positive unit preserving linear maps

$$T_1 \colon \mathcal{A} \to \mathcal{A} \quad , \quad T_2 \colon \mathcal{B} \to \mathcal{B} \, ,$$

there exists a (faithful) normal completely positive unit preserving linear map

$$T \colon \mathcal{A} \vee \mathcal{B} \to \mathcal{A} \vee \mathcal{B}$$

such that

$$T(A) = T_1(A) \, , \, T(B) = T_2(B) \, , \, T(AB) = T(A)T(B) \, ,$$

for all $A \in \mathcal{A}$, $B \in \mathcal{B}$.

Hence, for operationally independent subsystems, *any* operation performed on either subsystem is compatible with performing *any* operation on the other subsystem in the strong sense that *both* can be implemented by a single operation on the full system which factors across the subalgebras. Operational independence of $(\mathcal{A}, \mathcal{B})$ is strictly stronger than statistical independence [61], and we expect it to be logically independent of the commensurability of observables. In the next section we consider a yet stronger independence property.

4 The Split Property

We turn now to the split property, an important structure property of inclusions of von Neumann algebras, which has been intensively studied for the purposes of both abstract operator algebra theory and local quantum physics. We shall see that it provides a particularly useful formalization of subsystem independence and propose this as primary among notions of independence. In the following $\mathcal{A}\overline{\otimes}\mathcal{B}$ denotes the (unique W^*-) tensor product of two von Neumann algebras $\mathcal{A}$ and $\mathcal{B}$, which can be thought of as acting upon $\mathcal{H} \otimes \mathcal{H}$ in the natural manner $(A \otimes B)(\Phi \otimes \Psi) = A\Phi \otimes B\Psi$, for all $A \in \mathcal{A}$, $B \in \mathcal{B}$, and $\Phi, \Psi \in \mathcal{H}$. A von Neumann algebra $\mathcal{M}$ is a factor if its center $\mathcal{M} \cap \mathcal{M}'$ consists only of multiples of I. A factor is type I if it is isomorphic to $\mathcal{B}(\mathcal{K})$ for some Hilbert space $\mathcal{K}$. In general, a von Neumann algebra is type I if it is isomorphic to the tensor product $\mathcal{B}(\mathcal{K})\overline{\otimes}\mathcal{Z}$, where $\mathcal{Z}$ is an abelian (*i.e.* commutative) von Neumann algebra. Hence, abelian von Neumann algebras are type I [42, 67].

Split Property: A pair $(\mathcal{A}, \mathcal{B})$ of von Neumann algebras is *split* if there exists a type I factor $\mathcal{M}$ such that $\mathcal{A} \subset \mathcal{M} \subset \mathcal{B}'$.

Although according to the usage introduced in [30] we should say that the pair $(\mathcal{A}, \mathcal{B}')$ is split, it is for our purposes more convenient to use the terminology established above. It is immediately clear that mutually commuting type I factors are split. The split property is equivalent to a structure property which may be more familiar to the reader.

Theorem 4.1 ([5, 25]) *For a mutually commuting pair $(\mathcal{A}, \mathcal{B})$ of von Neumann algebras acting on a Hilbert space $\mathcal{H}$, the following are equivalent.*

1. *The pair $(\mathcal{A}, \mathcal{B})$ is split.*
2. *The map*

$$AB \to A \otimes B \, , \, A \in \mathcal{A} \, , \, B \in \mathcal{B}$$

extends to a spatial isomorphism of $\mathcal{A} \vee \mathcal{B}$ with $\mathcal{A}\overline{\otimes}\mathcal{B}$, i.e. there exists a unitary operator $U : \mathcal{H} \to \mathcal{H} \otimes \mathcal{H}$ such that $UABU^* = A \otimes B$ for all $A \in \mathcal{A}$, $B \in \mathcal{B}$.

If $(\mathcal{A}, \mathcal{B})$ is split, then it is operationally independent, thus also statistically independent, and $\mathcal{A} \subset \mathcal{B}'$, but the converse is false, *i.e.* the split property is strictly stronger than any combination of the other three [25, 61, 64].

Before we explore various operational consequences of the split property, let us examine the status of these notions of independence in relativistic quantum field theory. First of all, being spatiotemporally distinct does *not* entail any kind of independence of the corresponding observable algebras at all. In particular, even if $\mathcal{O}_1 \cap \mathcal{O}_2 = \emptyset$, all four independence properties can be false. This is not surprising in light of the causal propagation of influences in relativistic quantum field theory — cf. *e.g.* [59].

As mentioned above, if $\mathcal{O}_1$ and $\mathcal{O}_2$ are spacelike separated (this entails $\mathcal{O}_1 \cap \mathcal{O}_2 = \emptyset$), then $\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)'$ is satisfied in essentially all constructed quantum field models. Indeed, the property of locality is such a *sine qua non*, that it is normally posited as an axiom of AQFT. In addition, in typical models the pair $(\mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2))$ is statistically independent whenever $\mathcal{O}_1$ and $\mathcal{O}_2$ are spacelike separated [32]. However, when such spacelike separated regions are tangent, *i.e.* their closures have nonempty intersection, then $(\mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2))$ is neither operationally independent nor split [61, 63].

But when $\mathcal{O}_1$ and $\mathcal{O}_2$ are *strictly* spacelike separated, *i.e.* when $\mathcal{O}_1 + \mathcal{N}$ is spacelike separated from $\mathcal{O}_2$ for some neighborhood $\mathcal{N}$ of the origin in $\mathbb{R}^4$, then it is typically the case that the pair $(\mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2))$ is split. Indeed, the split property has been verified for all pairs associated with strictly spacelike separated (precompact, convex) regions $\mathcal{O}_1, \mathcal{O}_2$ in a number of physically relevant quantum field models, both interacting and noninteracting [5, 19, 25, 48, 62]. There do exist models in which the spacelike separation between $\mathcal{O}_1$ and $\mathcal{O}_2$ must exceed a certain minimum bound before the corresponding pair $(\mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2))$ is split (*distal split property*) [27], but the only known models in which even the distal split property does not hold are physically pathological models, such as models with noncompact global gauge group and models of free particles such that the number of species of particles grows rapidly with mass [30]. It should be remarked that in more than two spacetime dimensions, pairs $(\mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2))$ associated with certain spacelike separated regions which are unbounded (such as wedge regions) cannot be split, no matter how large the spacelike separation between them may be [5]. Nonetheless, under fairly general circumstances observable algebras associated with certain smaller but still unbounded regions called spacelike cones are split when the regions are strictly spacelike separated [26].

Prospective readers of the above–mentioned papers should note that the "split property" which is proven there is actually stronger than the "split property" we defined above. Indeed, in the first decades of the development of AQFT the property which is verified in the cited papers was called the *funnel property*. The terminology seems to be in flux now. In any case, we explain the connection between the funnel property and the property we term the split property. To minimize technical complications we consider only spacetime regions called double

cones. These are nonempty intersections of some forward light cone with some backward light cone [1, 36]. They are convex, precompact and satisfy $(\mathcal{O}')' = \mathcal{O}$. The funnel property obtains when for any two double cones $\mathcal{O}$, $\widetilde{\mathcal{O}}$ such that the closure of $\mathcal{O}$ is contained in $\widetilde{\mathcal{O}}$, there exists a type I factor $\mathcal{M}$ such that $\mathcal{A}(\mathcal{O}) \subset \mathcal{M} \subset \mathcal{A}(\widetilde{\mathcal{O}})$. Thus, the pair $(\mathcal{A}(\mathcal{O}), \mathcal{A}(\widetilde{\mathcal{O}})')$ is split in our sense. If $\mathcal{O}_1$ is strictly spacelike separated from $\mathcal{O}_2$, then there exists a double cone $\widetilde{\mathcal{O}}$ containing the closure of $\mathcal{O}_1$ such that $\mathcal{O}_2 \subset \widetilde{\mathcal{O}}'$. By locality, one has $\mathcal{A}(\mathcal{O}_2) \subset \mathcal{A}(\widetilde{\mathcal{O}})'$. The funnel property therefore entails that the pair $(\mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2))$ is split in our sense. The apparent loss of the distinguishing term funnel property is regrettable. Similarly, in the above papers the distal split property refers to the requirement that there is a minimal distance between the boundaries of $\mathcal{O}$ and $\widetilde{\mathcal{O}}$ before there exists a type I factor $\mathcal{M}$ such that $\mathcal{A}(\mathcal{O}) \subset \mathcal{M} \subset \mathcal{A}(\widetilde{\mathcal{O}})$.

In addition to *this* evidence that the split property is commonly satisfied by physically relevant quantum field models, there is further support for this hypothesis. The split property for all pairs $(\mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2))$ associated with strictly spacelike separated (precompact, convex) regions $\mathcal{O}_1, \mathcal{O}_2$ (in fact, the funnel property) has been shown [6,9,12] to be a consequence of a condition (nuclearity) which assures that the model is thermodynamically well–behaved (*e.g.* thermal equilibrium states exist for all temperatures [7, 10]). The nuclearity condition, which necessitates rather heavy technical baggage and will therefore not be explained here, expresses the requirement that the energy–level density for any states essentially localized in a bounded spacetime region cannot grow too fast with the energy. There is good physical reason to expect that physically relevant models will satisfy this condition and therefore also the split property [6, 12].

Although both the split property and the good thermodynamic behavior are consequences of the nuclearity condition (and not necessarily the converse[5]), the existence of a minimal spacelike separation for which the split property holds (the distal split property) has been shown to be related to the existence of a maximal temperature above which thermal equilibrium states do not exist [12, 13]. In addition, all known models which do not satisfy at least the distal split property have in common that they describe systems with an enormous number of local degrees of freedom. These systems, as a consequence, do not admit thermal equilibrium states, *e.g.* [7]. Hence, there are at least some indications that the split property is directly related in some not yet fully understood manner to good thermodynamic properties of quantum fields.

We have already pointed out the fact that commuting type I factors are always split; indeed, for type I factors $\mathcal{A} \subset \mathcal{B}'$ is *equivalent* to $(\mathcal{A}, \mathcal{B})$ is split. In nonrelativistic quantum mechanics the observable algebras are typically type I. Moreover, in the now extensive literature on quantum information theory (developed up to the present primarily for nonrelativistic quantum theory and quantum systems of only finitely many degrees of freedom) the word "subsystem" has become synony-

[5] However, in [11] the split property is proven to be equivalent to the nuclearity of a certain family of maps and certain other partial converses are established in [12].

mous with a type I observable algebra which is a factor in a tensor product of other "subsystems" — see *e.g.* [43]. Hence, the split property is the second of the above–mentioned two independence properties commonly used by theoretical physicists. However, it is employed by them in a setting where the most distinctive advantages of the property cannot reveal themselves.

5 Physical Characterizations of the Split Property

In light of the above, one may tentatively conclude that the split property obtains in some generality in physically relevant quantum field models. We further examine the warrant for this property by explaining some physically meaningful characterizations of the split property. We begin with one of the first found, which generalized a characterization proven in [8]. We present it in a form given in [64], since the original [70] requires the full apparatus of AQFT, the minimal assumptions of which entail that local algebras are "almost" type III [2]. (In fact, with some additional hypotheses — which nonetheless are also commonly fulfilled by most concrete models — local observable algebras *are* type III, see *e.g.* [34].) A characterizing property of type III factors (and the condition actually needed in the proof of the following theorem) is that in such an algebra $\mathcal{M}$ for any nonzero projection $P \in \mathcal{M}$ there exists a partial isometry $W \in \mathcal{M}$ such that $WW^* = P$ and $W^*W = I$. Thus all nonzero projections in type III algebras are infinite-dimensional [42, 67].[6]

Theorem 5.1 ([64, 70]) *Let $\mathcal{A}$, $\mathcal{B}$, be commuting type III von Neumann factors on the Hilbert space $\mathcal{H}$. Then the following are equivalent:*

1. The pair $(\mathcal{A}, \mathcal{B})$ is split.

2. Local preparability of some normal state: there exists a normal state ϕ and a normal positive map $T : \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ such that $T(A) = \phi(A)T(1)$ for all $A \in \mathcal{A}$ and $T(B) = T(1)B$ for all $B \in \mathcal{B}$.

3. Nonselective local preparability of all normal states: for any normal state ϕ there exists a map $T : \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ of the form $T(C) = \sum W_i^* C W_i$ with $W_i \in \mathcal{B}'$ such that $T(A) = \phi(A)T(1)$ for all $A \in \mathcal{A}$ and $T(1) = 1$.

So for any state ω and all $A \in \mathcal{A}$, $B \in \mathcal{B}$,

$$\begin{aligned} \omega(T(AB)) &= \omega(\sum W_i^* AB W_i) = \omega(B \sum W_i^* A W_i) = \omega(BT(A)) \\ &= \phi(A)\omega(B) \,. \end{aligned}$$

[6] The readers of this journal might find the introduction to types of von Neumann algebras given in [60] relatively painless.

$\omega \circ T$ is therefore a product state on $\mathcal{A} \vee \mathcal{B}$. Thus normal product states with arbitrary normal partial states can be locally prepared on $\mathcal{A} \vee \mathcal{B}$ if and only if $(\mathcal{A}, \mathcal{B})$ is split, whenever the algebras are type III.

In view of the previous discussion of operations, the operational content of conditions (2) and (3) in Theorem 5.1 should be clear. Note that the operations involved in the above theorem are solely operations to prepare states. The following result has been recently proven and involves, as discussed in Section 3, arbitrary operations.

Theorem 5.2 ([61]) *If the pair $(\mathcal{A}, \mathcal{B})$ is split, then it is operationally independent. Moreover, if either of the algebras is type III or a factor, then the pair $(\mathcal{A}, \mathcal{B})$ is operationally independent if and only if it is split.*

In fact, it is proven in [61] that the version of operational independence discussed here is equivalent to a property called W^*-independence in the product sense. And, as shown in [25], in a large number of circumstances (though not all) W^*-independence in the product sense is equivalent to the split property. The two particular circumstances mentioned in Theorem 5.2 are singled out, because they arise frequently in applications to quantum theory.

We turn next to another, relatively recently discovered characterization of the split property which has operational interpretation. A bit of background information might be useful here. As mentioned above, the commensurability of observables and statistical independence are independent properties. Although some rather *ad hoc* and nontransparent conditions expressed in terms of states on algebras $\mathcal{A}$, $\mathcal{B}$, are known which entail that $\mathcal{A} \subset \mathcal{B}'$ (see [64] for a discussion of these as well as references), workers in the field have long desired operationally meaningful conditions on the states on $\mathcal{A}$, $\mathcal{B}$, which would imply that the algebras mutually commute. This search was taken up in [20], but what was found was a characterization of the split property.

Let $\mathcal{A}$, $\mathcal{B}$, be two observable algebras which need not commute, and let $E \in \mathcal{A}$, $F \in \mathcal{B}$, be projections. Such projections represent "yes–no" observables, such as "the spin of the electron in the z-direction is up–down" or "the polarization of the photon is right–handed—left–handed". What kind of meaningful coincidence experiment can be designed in the case that $EF \neq FE$? Let $E \wedge F$ denote the largest projection in $\mathcal{B}(\mathcal{H})$ dominated by both E and F. Note that $E \wedge F = EF$ if and only if $EF = FE$. Moreover,

$$E \wedge F = \lim_{n \to \infty} (EF)^n = \lim_{n \to \infty} (FE)^n \, .$$

In a suitable coincidence experiment involving the observables E and F, a "yes" result in the apparatus should yield "yes" with certainty for any subsequent measurement of either E or F; moreover, the acceptance rate of the device should be maximized by the design of the experiment. A bit of thought convinces one that $E \wedge F$ is the (idealized) observable corresponding to this optimized coincidence apparatus.

Definition 5.3 *A state ω on $\mathcal{A}\vee\mathcal{B}$ is $\mathcal{A}$-$\mathcal{B}$–uncorrelated if $\omega(E\wedge F)=\omega(E)\omega(F)$ for all projections $E\in\mathcal{A}$ and $F\in\mathcal{B}$.*

Note that if $\mathcal{A}\subset\mathcal{B}'$, then in such an $\mathcal{A}$-$\mathcal{B}$–uncorrelated state one would have $\omega(EF)=\omega(E)\omega(F)$ for all projections $E\in\mathcal{A}$ and $F\in\mathcal{B}$. This would in turn entail that $\omega(AB)=\omega(A)\omega(B)$ for all $A\in\mathcal{A}$ and $B\in\mathcal{B}$ (by the spectral theorem and the fact that $\mathcal{A}$ and $\mathcal{B}$ are C^*-algebras). But this latter condition corresponds exactly to the notion of independence widely used in classical probability theory (and is a direct generalization of it, cf. *e.g.* [60]). However, the condition is also operationally meaningful when the algebras do not commute. Indeed, this fact allows one to define a quantitative measure of the degree of noncommutativity of two such algebras [20]. Instead, we return to our immediate purpose.

Theorem 5.4 ([20]) *Let $\mathcal{A}$, $\mathcal{B}$, be two von Neumann algebras which need not commute (and $\mathcal{A}\vee\mathcal{B}$ be a factor). Then $(\mathcal{A},\mathcal{B})$ is split if and only if there exists a normal $\mathcal{A}$-$\mathcal{B}$–uncorrelated state on $\mathcal{A}\vee\mathcal{B}$.*

The existence of a single normal $\mathcal{A}$-$\mathcal{B}$–uncorrelated state on $\mathcal{A}\vee\mathcal{B}$ suffices to entail that $(\mathcal{A},\mathcal{B})$ is split. However, if $(\mathcal{A},\mathcal{B})$ is split, then $\mathcal{A}\subset\mathcal{B}'$, so that $E\wedge F=EF$ for all projections $E\in\mathcal{A}$, $F\in\mathcal{B}$. In addition, for any normal states ϕ_1 on $\mathcal{A}$ and ϕ_2 on $\mathcal{B}$, there exists a normal state $\phi_1\otimes\phi_2$ on $\mathcal{A}\overline{\otimes}\mathcal{B}$ such that

$$(\phi_1\otimes\phi_2)(A\otimes B)=\phi_1(A)\phi_2(B)\,,$$

for all $A\in\mathcal{A}$ and $B\in\mathcal{B}$. Defining $\psi:\mathcal{A}\vee\mathcal{B}\to\mathbb{C}$ by

$$\psi(AB)=(\phi_1\otimes\phi_2)(UABU^*)$$

(and extending to $\mathcal{A}\vee\mathcal{B}$ by linearity and continuity), where U is the unitary from Theorem 4.1, one sees that

$$\begin{aligned}\psi(E\wedge F) &= \psi(EF)=(\phi_1\otimes\phi_2)(UEFU^*)\\ &= (\phi_1\otimes\phi_2)(E\otimes F)=\phi_1(E)\phi_2(F)\\ &= \psi(E)\psi(F)\,,\end{aligned}$$

for all projections $E\in\mathcal{A}$, $F\in\mathcal{B}$. In other words, if $(\mathcal{A},\mathcal{B})$ is split then any normal partial states on the subalgebras have extensions to normal $\mathcal{A}$-$\mathcal{B}$–uncorrelated states on $\mathcal{A}\vee\mathcal{B}$.

The three theorems in this section provide different operational characterizations of the split property. In the following section we shall discuss some physically relevant consequences of the split property with an eye towards those which shed further light upon its relevance to subsystems.

6 Further Consequences of the Split Property

In general, in relativistic quantum field theory one has global energy, momentum and charge observables (say Q) which have meaning for the full quantum system [1,36]. These cannot be localized in any region with nonempty causal complement and cannot directly refer to any subsystem. But to any subsystem worth the name one must be able to attribute such quantities. This is a highly nontrivial matter, but if the funnel property holds[7], then for any double cone $\mathcal{O}$ and any slightly larger double cone $\widetilde{\mathcal{O}}$ there exist observables in $\mathcal{A}(\widetilde{\mathcal{O}})$ (say $Q_{\widetilde{\mathcal{O}}}$)[8] which are indistinguishable from the corresponding global observables for any experiment implementable in $\mathcal{O}$:

$$e^{itQ}Ae^{-itQ} = e^{itQ_{\tilde{\mathcal{O}}}}Ae^{-itQ_{\tilde{\mathcal{O}}}}\,, \tag{6.3}$$

for all $A \in \mathcal{A}(\mathcal{O})$ and all $t \in \mathbb{R}$, if Q is a generator of a global gauge group, or for all t in a suitable neighborhood of 0 if Q is a global energy or momentum operator (see below) [8, 28–30].[9] In fact, if the global theory is supplied with a strongly continuous unitary representation $U(\mathcal{P}_+^\uparrow)$ of the identity component of the Poincaré group acting covariantly upon the observable algebras

$$U(\lambda)\mathcal{A}(\mathcal{O})U(\lambda)^{-1} = \mathcal{A}(\lambda\mathcal{O})\,,$$

for all $\lambda \in \mathcal{P}_+^\uparrow$ and $\mathcal{O}$, then for a fixed spacetime region $\mathcal{O}$, any neighborhood $\mathcal{P}_0$ of the identity in $\mathcal{P}_+^\uparrow$ and any region $\widetilde{\mathcal{O}}$ such that $\lambda\mathcal{O} \subset \widetilde{\mathcal{O}}$ for all $\lambda \in \mathcal{P}_0$, one has unitaries $U_{\widetilde{\mathcal{O}}}(\lambda) \in \mathcal{A}(\widetilde{\mathcal{O}})$ such that

$$U_{\widetilde{\mathcal{O}}}(\lambda)AU_{\widetilde{\mathcal{O}}}(\lambda)^{-1} = U(\lambda)AU(\lambda)^{-1}\,,$$

for all $\lambda \in \mathcal{P}_0$ and $A \in \mathcal{A}(\mathcal{O})$. In addition, the local implementers $Q_{\widetilde{\mathcal{O}}}$ have the *same* spectrum as their global counterparts [8]. For example, if the generators of the translation subgroup $U(\mathbb{R}^4) \subset U(\mathcal{P}_+^\uparrow)$ (which have the interpretation of the global energy–momentum observables of the quantum field theory) satisfy the relativistic spectrum condition, then so do the generators of the local implementers $U_{\widetilde{\mathcal{O}}}(x)$, $x \in \mathbb{R}^4$. Hence, subsystems whose observables are localized in $\mathcal{O}$ can be attributed localized energy–momentum and charge operators, at the minor cost of accepting a slightly larger localization region for the latter.[10] The arguments yielding these results can also be applied to supersymmetric theories and theories in which topological charges are present [8].

[7]As mentioned above, also strictly spacelike separated spacelike cones are associated with split observable algebras in some generality. For this reason, the results discussed here are also valid for charges which can only be localized in such spacelike cones and not in bounded regions, as is expected in massive gauge theories [26].

[8]If Q is an unbounded selfadjoint operator, then so is $Q_{\widetilde{\mathcal{O}}}$, which entails $Q_{\widetilde{\mathcal{O}}} \notin \mathcal{A}(\widetilde{\mathcal{O}})$. Nonetheless, $\mathcal{A}(\widetilde{\mathcal{O}})$ does contain all of the spectral projections of $Q_{\widetilde{\mathcal{O}}}$.

[9]For the reader who knows that the action of gauge groups upon the observables is trivial, we note that the same relation (6.3) holds when the net of observable algebras $\mathcal{O} \mapsto \mathcal{A}(\mathcal{O})$ is replaced by a net of field algebras $\mathcal{O} \mapsto \mathcal{F}(\mathcal{O})$ [8], upon which the action *is* nontrivial.

[10]In light of the observation made above that there is no operational way to determine the minimum spacetime localization of an observable, this price is minor, indeed.

We turn finally to some significant consequences of the split property which are not immediately relevant to subsystem independence, but which will be very briefly mentioned to further illustrate to the reader the mathematical and conceptual advantages of the split property. We do not attempt to provide an exhaustive list of the known or suspected consequences of this kind.

In general, in relativistic quantum field theory quantities such as entropy, entanglement of formation and relative entropy of entanglement are infinite (undefinable). If the funnel property (or nuclearity, which entails the funnel property) holds for strictly spacelike separated regions, then such quantities can be given sensible (strictly positive and finite) meaning at least for a dense subset of normal states [52, 53].

As mentioned above, the local algebras in AQFT are typically type III (in fact, type III_1 in the classification of A. Connes [34]). However, there exist nonisomorphic type III_1 algebras. A further consequence of the funnel property is that essentially *all* local observable algebras are isomorphic [9]. This makes it unmistakeably obvious that it is not the algebras of observables themselves in which are encoded most of the physical information of the theory but rather the *inclusions* of the local algebras, *i.e.* how the algebra $\mathcal{A}(\mathcal{O}_1)$ sits inside the algebra $\mathcal{A}(\mathcal{O}_2)$ when $\mathcal{O}_1 \subset \mathcal{O}_2$.

It has also emerged from recent work [18, 19, 48, 49] that the funnel property is exceedingly useful in the rigorous construction of quantum field models using algebraic methods. For a relatively nontechnical overview of the results obtained with these methods, as well as further references, see [66].

The split property can also be employed to resolve other conceptual problems in quantum field theory. An example is the fairly recent exchange in which G.C. Hegerfeldt argued that there are causality problems in Fermi's classic two–atom system [39]. But D. Buchholz and J. Yngvason convincingly retorted that Hegerfeldt's argument rested upon his tacit use of a local, minimal projection.[11] But local observable algebras $\mathcal{A}(\mathcal{O})$ are type III algebras, which contain no minimal projections. (Indeed, as mentioned above, in type III algebras all nonzero projections are mutually equivalent in a sense which places them as far away from minimal projections as is possible in the theory of operator algebras.) On the other hand, the type I algebras which interpolate between local algebras in the split property *do* have minimal projections. Buchholz and Yngvason explain how once this point is taken into account, the causality problems Hegerfeldt described evaporate [16].

[11] A minimal projection of a von Neumann algebra $\mathcal{M}$ is a projection in $\mathcal{M}$ which dominates no other projection in $\mathcal{M}$ other than the zero operator.

7 Concluding Remarks

We conclude that it is meaningful to speak of independent subsystems in relativistic quantum theory, *if* they can be localized in spacetime regions $\mathcal{O}_1$, resp. $\mathcal{O}_2$, such that $\mathcal{A}(\mathcal{O}_1)$ and $\mathcal{A}(\mathcal{O}_2)$ satisfy the split property. For then their observables are mutually commensurable, they can be independently and locally prepared in arbitrary states, they are "operationally independent," and they possess mutually compatible localized energy, momentum and charge observables, to mention just a few desirable properties. In addition, the split property for spacetime regions which are (sufficiently) strictly spacelike separated holds widely in the quantum field models constructed up to this point and is expected to hold in most, if not all, physically relevant models to be constructed in the future. Syllogistically, it is therefore meaningful to speak of independent subsystems in relativistic microscopic physics.

These observations are, however, only a starting point for the examination of the notion of such subsystems. No attempt has been made here to provide a definition of microscopic subsystem, although it is clear that this is an interesting philosophical question. We briefly indicate some of the complexities involved in coming to grips with this question in relativistic quantum physics.

To begin, let us consider the notion of quarks, which are essential to the understanding of the physics of strong interactions in the Standard Model of elementary particle physics. Although there are indirect indications that something like quarks exist within baryons such as protons and neutrons, can one correctly speak of them as subsystems? According to heuristic computations made in the Standard Model, quarks can never be observed as isolated objects, since the force of attraction between coupled quarks grows with their separation. This is antithetical to the usual situation one faces when analyzing nature into subsystems, where the interparticle forces decrease as the systems are mutually separated. The quarks which are imagined to be bound together in colliding baryons can exchange partners but, apparently, cannot be isolated as independent subsystems, since each quark is always in intense interaction with at least one other quark. Are such objects subsystems? Are they more than convenient theoretical constructs? Do subsystems need to be "real"?

These questions seem to us to be nontrivial. We remark that quarks can possibly be understood in an intrinsic, objective manner as ultraparticles [14,15,17], but it is not clear that this (theoretically significant) observation diminishes the burden of the above–mentioned philosophical questions.

To indicate further the nontrivial nature of the question at hand, we turn to the notion of "particle" in relativistic quantum field theory, a subject receiving increasing attention from philosophers of physics. Though some of these have chosen to deny the existence of such particles (*e.g.* [33]) and others have argued, with reservations, in their favor (*e.g.* [37]), no one can deny their centrality in the discourse and *Weltanschauung* of quantum field theorists. Certainly, it is

already evident that the comfortable notion of particles familiar from classical theory is quite out of place in relativistic quantum physics; this is true even before more subtle notions of particles which are in use among mathematical physicists, such as infraparticles and ultraparticles, have even entered the discussion among philosophers of physics.

But *something* is being observed at large times and distances (relative to collision times and distances) in scattering experiments in CERN and elsewhere which acts much like particles should, and whether one views these as particles, local excitations of the quantum field or some other alternative, one must ask if these can and should be viewed as (independent) subsystems. Certainly, elementary particle physicists are acting as if they were. But it has been rigorously established in AQFT that, whatever these things are, they (or, more precisely, the idealized apparata which count them) are not strictly localized, *i.e.* cannot be elements of a local algebra $\mathcal{A}(\mathcal{O})$ with $\mathcal{O}' \neq \emptyset$ [1, 36]. All is not lost, however, since they can be arbitrarily well approximated in norm by strictly localized observables [1, 15, 36] (see also [37] for an explication written for philosophers of physics), but this is an additional complication in the problem of deciding if these particle–like objects can be viewed as independent subsystems, even asymptotically at large times and distances.

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